Class Problem 6.1

Crude oil is carried by pipelines from oil fields and storage areas over hundreds of miles to urban and industrial centers. Output is the amount of oil carried per day, and the two principal inputs are pipeline diameter and the horsepower applied to the oil carried. Suppose that an oil company estimates that throughput per day in its pipeline is given by

$$Q = 286H^{0.37},$$

where H is horsepower usage, which costs \$30 per unit, and throughput has a marginal revenue of \$2 per unit. The marginal product of horsepower is

$$MP_H = 105.82H^{-0.63},$$

the marginal revenue product is

$$MRP_H = 211.64H^{-0.63}.$$

Next, we set MRP_H equal to the marginal expenditure of horsepower, \$30; thus,

$$211.64H^{-0.63} = 30 \Rightarrow H^{-0.63} = 0.14175.$$

Thus the optimal level of horsepower usage (which maximizes profit) is $H = 22.22$.

Extra Credit:

1. Suppose that the oil company's pipeline operation has total daily fixed costs of \$1,000. If the optimal level of horsepower usage is chosen, what is the maximum daily profit that the firm can earn from its pipeline operation? (hint: note that since the market for oil is perfectly competitive, this implies that $P = MR$).

   Solution: Since the optimal level of horsepower usage is $H = 22.22$, this means that $Q = 286H^{0.37} = 286(22.22)^{0.37} = 900.87$ units of crude oil per day. Since marginal revenue of \$2 per unit and the market is perfectly competitive, this implies that $P = MR$; therefore, total daily revenue = TR = 900.87(\$2) = \$1,801.74. Furthermore, since the cost per unit of horsepower usage is \$30, this implies that total daily cost is TC = \$1,000 + \$30(22.22) = \$1,666.60. Thus the maximum daily profit that this firm can expect to earn its pipeline operation is $\pi = TR - TC$ = \$1,801.74 - \$1,666.60 = \$135.14.

2. Show that marginal revenue (MR) is equal to marginal cost (MC) in this problem. (hint: keep in mind that the marginal expenditure of horsepower, $ME_H = MC(MP_H)$ = \$30).

   Solution: In this problem, throughput has marginal revenue (MR) of \$2 per unit. The definition for the marginal revenue product of horsepower is $MRP_H = MR(MP_H)$ and the definition for the marginal expenditure of horsepower is $ME_H = MC(MP_H)$. Since we found the optimal level of horsepower usage by setting $MRP_H = ME_H$, if we divide both sides of this equation through by

MP_H, this implies that $MR = MC$. Since $MR = \$2$ in this problem, this means that MC is also equal to \$2.

Another way to show that $MR = MC = 2$ involve calculating the marginal product of horsepower directly. Since $MP_H = 105.82H^{-0.63}$ and $H = 22.22$, this implies that $MP_H = 105.82(22.22^{-0.63}) = 15$. Furthermore, since $ME_H = MC(MP_H) = 30$, this implies that $30 = MC(15) \Rightarrow MC = \$2$.